

## Problème

One considers the simple model of task delegation : the principal wants to delegate a task to an agent. the agent has to produce a good (quantity will be noticed  $q$ ) . This costs  $\theta q$  to him with  $\theta \in \{\underline{\theta}, \bar{\theta}\}$  with  $\underline{\theta} < \bar{\theta}$ . The principal extracts a surplus  $S(q)$ .  $S$  being concave and increasing. The principal proposes a contract, (or a menu of contracts)  $(t, q)$  where  $t$  is the payment made to the agent against a quantity  $q$ . If he refuses the contract, the agent obtains (with an outside option) a profit  $U_0(\theta)$ . That means that an agent with type  $\theta$  will accept only if his profit is higher than  $U_0(\theta)$ . In the course  $U_0(\theta) = 0$ .

### Part I

## perfect information with zero outside option (2points)

Assume first that the principal knows  $\theta$ .

1. What are the (First Best) optimal contracts  $(\bar{t}^*, \bar{q}^*)$  and  $(\underline{t}^*, \underline{q}^*)$  when  $\forall \theta, U_0(\theta) = 0$ .
2. What is the level of utility achieved by the two types of agents

### Part II

## Asymmetric information with zero outside option (4 points)

Assume that the principal does not observe  $\theta$  but knows the probability  $\nu$  that  $\theta = \bar{\theta}$  . We note (as usual) the contracts  $(\bar{t}, \bar{q})$  and  $(\underline{t}, \underline{q})$ .

3. Recall the participation and incentive constraints.
4. Compute the optimal second best contracts  $(\bar{t}^{SB}, \bar{q}^{SB})$ ,  $(\underline{t}^{SB}, \underline{q}^{SB})$ , and the profits  $(\bar{U}^{SB}, \underline{U}^{SB})$  achieved by the two types of agents.

(see next page, please....)

## Part III

# Non zero outside option (10 points)

We assume now that  $U_0(\bar{\theta}) = 0$  but  $U_0(\underline{\theta}) = \underline{u} > 0$ . That means that the good agent can obtain a “good” contract elsewhere where he can obtain  $\underline{u}$ .

We are still in the asymmetric information case.

5. What are the new incentive and participation constraints?(1 point)
6. What happens if  $\underline{u} \leq \underline{U}^{SB}$ ? (2 points, that is a gift)
7. What is the optimal contract if  $\underline{U}^{SB} < \underline{u} \leq (\bar{\theta} - \underline{\theta})\bar{q}^*$ ? (Hint : use a  $(q, t)$  picture with iso profit lines) (2 points)
8. What happens if  $(\bar{\theta} - \underline{\theta})\bar{q}^* < \underline{u} \leq (\bar{\theta} - \underline{\theta})\underline{q}^*$ ? (1 point)

Assume now that  $\underline{u} > (\bar{\theta} - \underline{\theta})\underline{q}^*$

9. Show that the bad agent could have an incentive to lie. (2 points)
10. What is the second best contract in that case? (2 points)

## Questions (4 points)

1. In the standard moral hazard model, when the agent is risk-neutral, show that optimality needs a negative payment in case of bad result.
2. In the standard moral hazard model, why, then, limited liability is a bad news for the principal?